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Limiting Performance of Massive MIMO Downlink Cellular Systems

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***Abstract*— Massive MIMO (multiple-input multiple-output) antenna technology can provide significant performance improvement for cellular systems in terms of both throughput and energy efficiency. It is widely recognized that inter-user interference can be eliminated with a large number of antennas because of the asymptotical orthogonality among users when linear MF (Matched Filter) downlink precoding is used in the eNodeB. In this paper, system level simulation performance for the downlink, based upon current LTE systems, provides an indication of the achievable potential system performance improvement by employing massive MIMO in future (5G) cellular networks. System simulations consider various performance limiting factors including non-ideal asymptotical orthogonality, transmit channel correlation, imperfect channel estimation and pilot contamination. It is demonstrated that even as the number of transmit antennas is increased to a large number, the accumulative effects of the non-ideal asymptotical orthogonality among different users constitute a significant limiting degradation to the system performance. Likewise, a significant performance decrease is also incurred due to transmit channel correlation, imperfect channel estimation, and pilot contamination.**

***Keywords*—Massive MIMO, pilot contamination, asymptotical orthogonality, transmit channel correlation, imperfect channel estimation.**

I. INTRODUCTION

Massive MIMO (multiple-input multiple-output) wireless technology uses a very large number of antennas with an order of magnitude more antennas than current LTE systems and is a leading candidate for inclusion in 5G systems. This will offer significant improvements in both the throughput and energy efficiency. As the number of antennas increases without limit, it is shown that the effects of uncorrelated noise and small-scale fading can be removed completely. That means when a large number of user terminals are scheduled simultaneously over the same physical layer resource, the channels of all the terminals are asymptotically orthogonal to each other, and both intra-cell and inter-cell user interference can be eliminated completely through linear signal processing methods, e.g., MF (Matched Filter) precoding or MF detection for the downlink and uplink respectively. That also means each user will be presented with a flat fading channel, so that each user may be scheduled with the full bandwidth, and the MAC (Media Access Control) resource allocation scheduling can be simplified and no control signaling of physical layer resource allocations will be required [1]–[3].

For classical MIMO systems, downlink precoding requires the CSI (channel state information) at the base station to determine the precoding matrix. To acquire the CSI at the base station in FDD systems, the base station transmits the pilot symbols in each transmit antenna port and the terminal feeds back the CSI to the base station. Correspondingly, the required number of pilot resources is proportional to the number of transmit antenna ports. As the number of the base station antennas increases without limit in massive MIMO systems, the required number of pilot resources will become unlimited, which is not feasible in practical FDD systems. For TDD systems the base station can acquire the CSI through uplink pilot symbols because of the channel reciprocity in TDD systems. Since the user terminals are typically equipped with a small number of transmit antennas, the required number of uplink pilot resources will be small and similar to that of classical MIMO systems. Because of this, massive MIMO is typically envisioned for use in TDD systems [1], [4], [5].

Owing to the limited pilot physical resources, users in a set of cells share the pilot resources. When a pilot sequence is used by a user in a cell and transmitted to its serving base station, the base station estimates the uplink channel, and the pilot sequence is interfered with by users that use the identical pilot sequence in neighboring cells. Based upon the contaminated uplink pilot channel, the base station conducts beamforming to transmit signals not only to its own users, but also does beamforming to users in neighboring cells, an effect well known as pilot contamination effects in massive MIMO systems [1], [2].

While massive MIMO theoretically eliminates many traditional research issues, e.g., eliminating inter-user interference, it introduces entirely new problems that need to be addressed e.g., reciprocity calibration, channel characterization, pilot contamination and implementation complexity in both the computation and data buffer [2], [5]. Furthermore, as will be addressed in this paper when massive MIMO is realized in practical systems, those seemingly irrelevant problems may constitute significant performance degradation.

In this paper, these basic massive MIMO problems that potentially degrade system performance are addressed including non-ideal asymptotical orthogonality, transmit channel correlation, imperfect channel estimation and pilot contamination effects. System level simulation performance of massive MIMO that is based upon current LTE systems, which is not found in the current literature, is provided taking into account those basic performance-limiting factors. This analysis provides insight on the potential system performance that can be achieved by using massive MIMO. Our expectation is that these results will provide further research motivation to efficiently solve these problems.

The rest of the paper is organized as follows. In Section II, massive MIMO systems are described. Section III presents several basic limiting factors for massive MIMO system performance that are addressed in the system level simulation. Section IV presents the system simulation setup and simulation results. Finally, our conclusions are presented in Section V

II. System Description

This paper considers a massive MIMO wireless cellular system with L cells denoted as $1,2 \ldots L$ respectively. Each cell consists of one base station with M antennas and K users equipped with only one antenna. The K users in each cell use orthogonal pilot resources at the base station to acquire the CSI, but share the same pilot resources with one of other users in each of the neighboring $L-1$ cells.

A. Signal Model

The base station j transmits a $M * 1$ precoded vector S_{fj}. The subscript f means forward link. The user k in the base station l receives the signal from the transmitted vectors from all the base stations, which is written as:

$$X_{kl} = \sqrt{\rho_f} \sum_{j=1}^{L} \mathrm{G_{jkl}}\, \mathrm{S_{fj}} + W_{kl} \tag{1}$$

where ρ_f is the signal-to-noise ratio for the forward link, W_{kl} is the complex independent and identically distributed (i.i.d.)

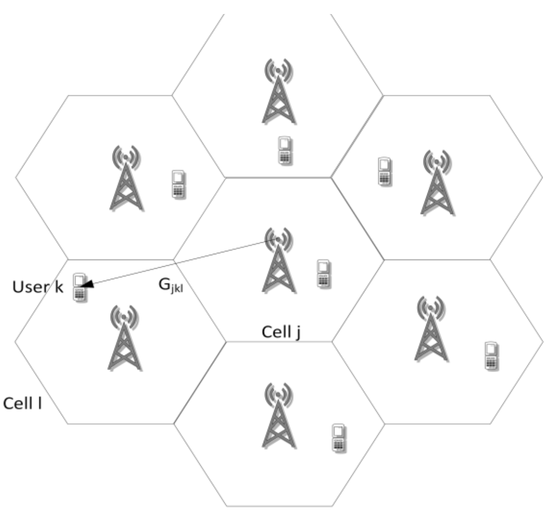

Fig.1. G_{jkl} is channel matrix between the user k in the base station l and M antennas in the base station j.

white Gaussian noise, and G_{jkl} is the $1 * M$ channel matrix between the user k in the base station l and M antennas in the base station j (see Fig. 1). The symbol G_{jkl} comprises the small-scale Rayleigh fading factors H_{jkl} and a large-scale factor $\beta_{jkl}^{1/2}$ that accounts for distance dependent attenuation and shadow fading and is regarded as the same for all the M antennas in a base station [1], where

$$G_{jkl} = H_{jkl}\beta_{jkl}^{1/2}\ . \tag{2}$$

Assume linear MF precoding is used in the base station the base station transmits

$$\mathrm{S_{fj}} = \hat{G}_{jj}^{*} a_j \tag{3}$$

where $\hat{G}_{jj}$ is the estimated $K * M$ channel matrix determined from the uplink pilot signals for the K users in the base station j that are contaminated by the identical pilot signals from users in other cells. The superscript $*$ denotes the conjugate transpose. The symbol a_j is the transmitted symbols for the K users in the base station j.

The estimated $K * M$ precoding matrix in the base station j

$$\hat{G}_{jj}^{*} = \sqrt{\rho_p} G_{jj}^{*} + \sqrt{\rho_p} \sum_{i \neq j} G_{ij}^{*} + V_j^{*} \tag{4}$$

where G_{jj} is the $K * M$ uplink channel matrix for the K users in the base station j, G_{ij} is the $K * M$ uplink channel matrix from the K users in the base station i to the base station j, ρ_p is the uplink pilot signal-to-noise ratio, V_j is the $1 * M$ i.i.d. white Gaussian noise samples.

Substituting (4) into (1), the received signal for the user k in the base station l is:

$$\begin{aligned} X_{kl} &= \sqrt{\rho_f} \sum_{j=1}^{L} \mathrm{G_{jkl}} \left(\sqrt{\rho_p} \sum_{i} G_{ij}^{*} + V_j^{*} \right) a_j + W_{kl} \\ &= \sqrt{\rho_f} \sum_{j=1}^{L} \mathrm{G_{jkl}} \Bigg(\sqrt{\rho_p} \sum_{i} \sum_{k} G_{jki}^{*} \\ &\quad + V_j^{*} \Bigg) a_{kj} + W_{kl} \end{aligned} \tag{5}$$

where G_{jki} is the $1 * M$ channel matrix between the user k in the base station i and M antennas in the base station j, and a_{kj} is the transmitted symbol for user k in the base station j.

As the number of antennas M is increased to infinity, the channels will be orthogonal to each other, since according to random matrix theory [6]:

$$\mathrm{G_{jkl}} G_{jki}^{*} \rightarrow 0 \text{ when } i \neq l \tag{6}$$

$$\mathrm{G_{jkl}} G_{jki}^{*} \rightarrow M\beta_{jkl} \text{ when } i = l \tag{7}$$

The received signal of the user k in the base station l becomes:

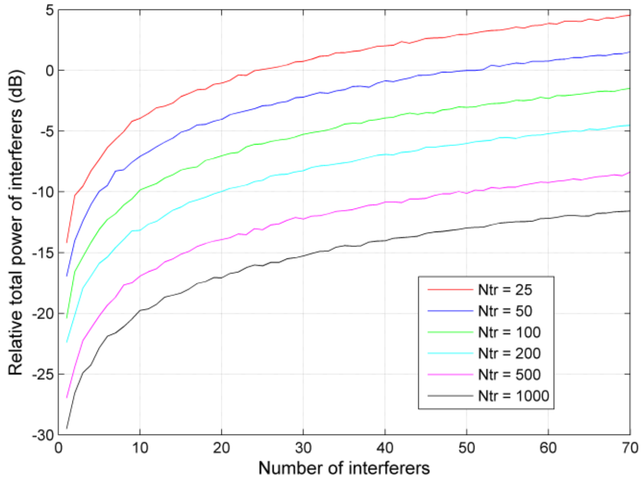

Fig.2. Relative power of interferers as a function of the number of interferers for 25, 50, 100, 200, 500 and 1000 transmit antennas.

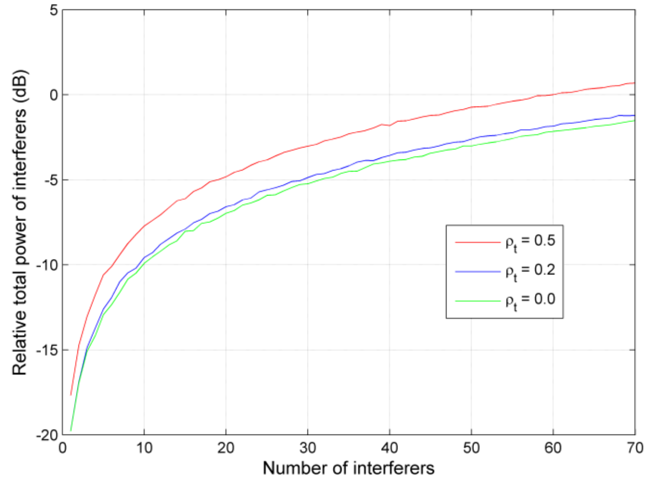

Fig.3. Relative power of interferers as a function of the number of interferers under different transmit correlation coefficients.

$$X_{kl} = \sqrt{\rho_f}\sqrt{\rho_p}M\beta_{lkl}a_{kl} + \sqrt{\rho_f}\sqrt{\rho_p}M\sum_{j\neq l}\beta_{jkl}\,a_{kk} + W_{kl} \tag{8}$$

The SIR (signal-to-interference ratio) of the user k in the base station l becomes:

$$SIR_{kl} = \frac{\beta_{lkl}^2}{\sum_{i\neq l}\beta_{ikl}^2} \tag{9}$$

In (9), the small-scale fading effects disappear because of the orthogonal effects (6)-(7). However, the pilot contamination effects are still present. Equation (9) shows that increasing the transmit power of the pilot symbols doesn't increase the SIR.

B. ZF (Zero forcing) Precoding

Compared with the MF precoding in (3), ZF precoding doesn't depend upon the channel independence between different users to eliminate inter-user interferences as (6)-(7) show. The ZF precoding technique uses the pseudoinverse of the channel matrix as the precoding matrix, and is given as (10) [7]:

$$\mathrm{S_{fj}} = \hat{G}_{jj}^*(\hat{G}_{jj}\hat{G}_{jj}^*)^{-1}a_j \tag{10}$$

where the parameters in (10) mean the same as those in (3), and the superscript -1 denotes the inverse of a matrix.

A problem with ZF precoding is that the computation of the pseudoinverse of the channel matrix requires the inversion operation of a $K*K$ matrix, which is computationally expensive. Moreover, ZF precoding will also greatly reduce the power of the useful signal while entirely eliminating inter-user interference when the user channels between users are highly correlated, an effect also known as noise enhancement

III. Basic Limiting Factors on System Performance

A. Non-Ideal Asymptotical Orthogonality

As (6) shows, according to random matrix theory, when the number of antennas approaches infinity, the channel correlation between different users will approach zero. This is one of the most attractive features of massive MIMO---the elimination of inter-user interference by only using simple linear MF precoding. However, in practical scenarios, the number of antennas will of course be limited. Thus residual inter-user interference will exist. As the number of multiplexed users increases, the accumulative inter-user residual interference will increase and may be comparable to the power of the concerned user. This will greatly impact the system performance. In Section IV, the system level performance for different numbers of transmit antennas will be simulated to give the quantitative effects of non-ideal asymptotical orthogonality on system performance. Figure 2 shows the average inter-user residual interference as a function of the number of multiplexed users with i.i.d. white Gaussian channels when 25, 50, 100, 200, 500 and 1000 antennas are assumed and linear MF precoding is used. From Fig. 2, we can observe that the residual power of interfering users is still comparable to the power of the concerned user when the number of transmit antennas is increased to 200 (about -5 dB) and the number of multiplexed users is increased to a relative large value (that is, it is possible to achieve a higher multiplexing gain and system throughput). This shows the inter-user residual interference cannot be neglected and will pose a significant degradation to system performance even when the number of transmit antennas are large, as will also be verified by the system level simulation in Section IV.B.

Although ZF precoding can remove the inter-user interference completely without dependence on the asymptotical orthogonality in massive MIMO, but a substantial complexity increase will be incurred, and the big advantage inherent in the massive MIMO that the inter-user interferences will be eliminated only through the simple MF precoding will not be made user of. So, in practical scenarios, a tradeoff between the performance and computation complexity should be considered. Section IV.B also gives the system performance of ZF precoding as a reference comparison with that of MF precoding.

B. Channel Correlation

Equations (6)-(7) assume spatially independent channels among the antennas. As the number of antennas becomes large, and their spacing reduced, the assumption of spatially independent channels among antennas is very difficult to be maintained. Hence the performance simulation of massive MIMO systems with correlated channels need to be performed and analyzed for practical applications of massive MIMO systems.

The spatially correlated channels obey the well-known correlation model [8]:

$$\mathrm{H} = R_r^{1/2} H_W R_t^{1/2} \tag{11}$$

where R_r and R_t denote the receive and transmit correlation matrix respectively, and H_W is the i.i.d. white Gaussian channel.

In this paper, we assume each user is equipped with only one antenna, then $R_r = 1$ and the element $r_{i,j}$ of R_t is exponentially correlated [9]:

$$r_{i,j} = {\rho_t}^{|i-j|} \tag{12}$$

where $\rho_t \in [0,1]$ is the transmit correlation coefficients.

Figure 3 shows the average inter-user residual interference as a function of the number of multiplexed users under different transmit correlation coefficients of 0.0, 0.2 and 0.5 respectively for 100 transmit antennas when linear MF precoding is used. From Fig. 3, we can observe the residual relative power of interfering users to the power of the concerned user is higher than 0 dB when the transmit correlation coefficient is increased to 0.5, and the number of multiplexed users is increased to a relative large value. This shows the inter-user residual interference is significant and sensitive to the transmit correlation if only linear MF precoding is used, as will also be verified by the system level simulation in Section IV.C.

C. Imperfect Channel Estimation

When the base station has the imperfect CSI, the MMSE (minimum mean square error) estimation of the channel matrix G_{jkl} is [2]:

$$\hat{G}_{jkl} = \xi G_{jkl} + \sqrt{1-\xi^2}E \tag{13}$$

where E is the $1 * M$ i.i.d. white Gaussian noise with power of 1 and ξ is the ratio of signal power to total power (signal power plus the noise power), that is:

$$\xi = \frac{\rho_f \beta_{jkl}}{\rho_f \beta_{jkl} + 1} \tag{14}$$

The simulation in Section IV.D will quantitatively provide the performance degradation by the pilot contamination effects.

D. Pilot Contamination

When one user in each cell transmits the same pilot sequence, and all the users in the same cell use the orthogonal pilot sequences, as (9) shows, pilot contamination effects will bring performance degradation and cannot be remediated by increasing pilot power. The simulation in Section IV.E will quantitatively provide the performance degradation by the pilot contamination effects.

TABLE I SYSTEM SIMULATION CONFIGURATION

Parameters	Assumption
Cellular layout	Hexagonal grid, 7 cell sites, 1 sector per site wrap around
Cell radius	500 meters
Path loss model	3GPP 36.942 urban model
Lognormal Shadowing	Fading mean: 0 dB Standard deviation: 10 dB Shadowing corrlelation between sites: 0.5
Antenna pattern	Omni-directional
eNodeB antennas	ULA 25, 50, 100 antennas Correlation coefficients: 0.0, 0.2 and 0.5
UE antennas	1 antenna
Carrier frequency/Duplex mode	TDD 2GHz
System bandwidth	20 MHz
Channel model	ITU Typical Urban (TU)
Receiver noise figure	9 dB
UE speed	30 km/h
Total BS TX power	46 dBm
UE power	23 dBm
Number of UEs	10 full buffer UEs in each cell
Scheduler	All-user Full bandwidth scheduling

IV. SYSTEM LEVEL SIMULATION RESULTS

A. System Level Simulation Setup

The system level simulation was run using Matlab [10]. The system simulation configuration is partly based upon LTE macro-cell system simulation baseline parameters [11] as shown in Table I. Seven omni-directional sites are simulated with 10 single-antenna UEs in each site equipped with 25, 50, and 100 transmit antennas with ULA (Uniform Linear Array) configurations respectively. The path loss model of 3GPP 36.942 urban model in used [12]. The TDD duplex mode is assumed, where the downlink channel matrix can be obtained through TDD channel reciprocity from the uplink channel matrix. A system bandwidth of 20 MHz and all-user full bandwidth scheduling are used, which means all 10 users in each cell are scheduled at the same time to the full bandwidth. In the simulation, for simplification of illustration, we assume

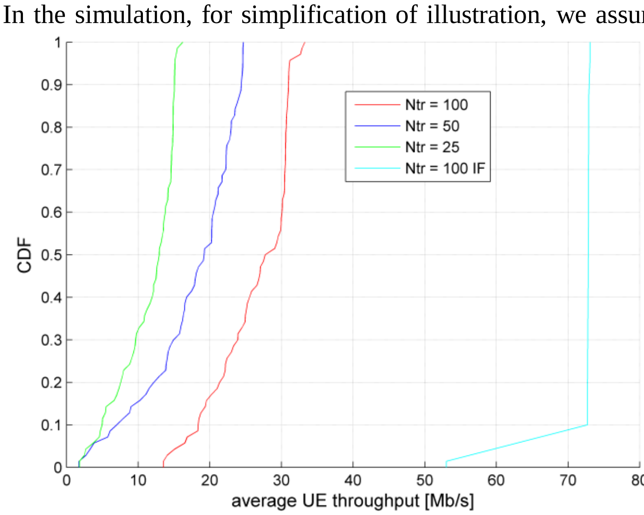

Fig.4. UE throughput CDF for different number of transmit antennas.

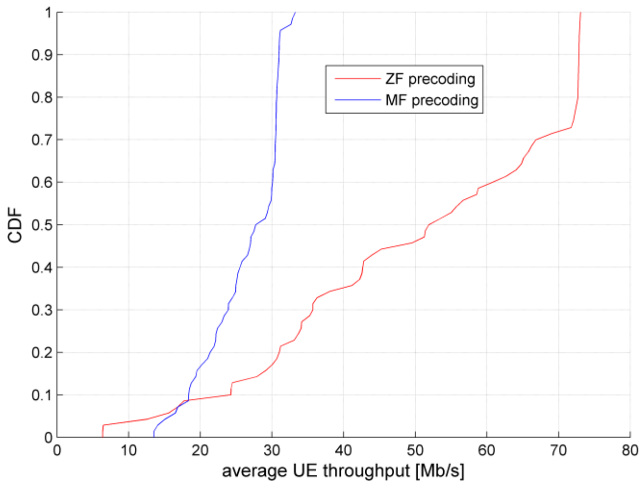

Fig.5. UE throughput CDF for 100 transmit antennas for ZF and MF precoding.

that all the system bandwidth is available for downlink data transmission in each subframe. The net system throughput for a specific TDD uplink-downlink configuration [13] can be easily derived. In the simulation downlink MF precoding is utilized, unless otherwise noted.

B. *Non-Ideal Asymptotical Orthogonality*

Figure 4 shows the UE throughput CDF (Cumulative Distribution Function) for different number of transmit antennas of 25, 50 and 100 respectively in each eNodeB. For comparison, system performance of an idealized interference free (IF) system of 100 antennas is also given. The IF system means channels between users are completely orthogonal, and inter-user interference is entirely eliminated through MF downlink precoding. From Fig. 4, we see that as the number of transmit antennas increases from 25 to 50 to 100, a 50% UE throughput increase from 13 to 19 to 28 Mbps can be achieved. But a larger performance gap can be observed between the IF system and systems with interference, both with 100 antennas. It is also apparent that with as many as 100 transmit antennas, though the inter-user interference from a single user is not significant because of their asymptotical orthogonality, the accumulated inter-user interference will be substantial, and will

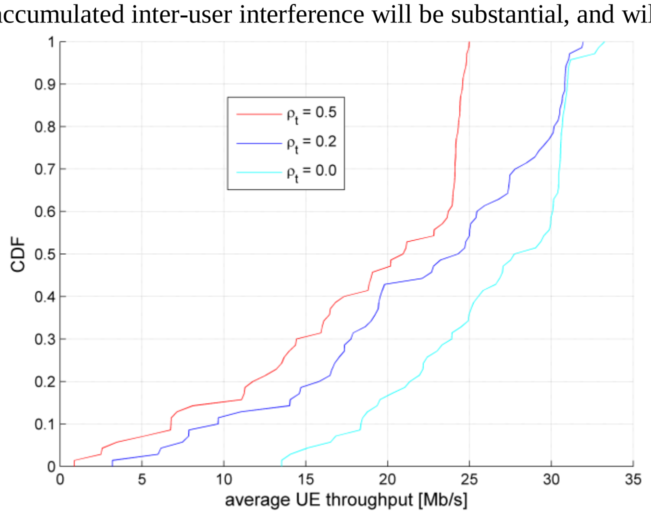

Fig.6. UE throughput CDF for 100 transmit antennas under different transmit correlation coefficients.

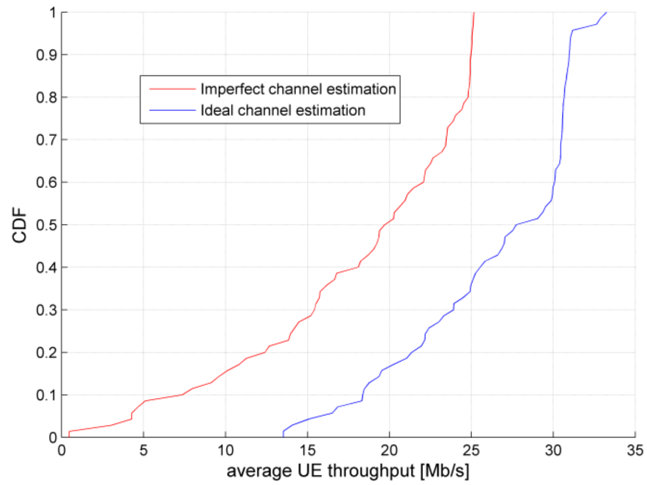

Fig.7. UE throughput CDF with imperfect channel estimation for 100 transmit antennas

constitute a significant performance degradation, and limitation, to the system.

One of the solutions to this problem is using other precoding technologies, e.g., ZF precoding. As shown earlier, ZF precoding is more efficient in eliminating inter-cell interference than MF precoding, but at the expense of much higher processing complexity and potentially enhanced noise power. Figure 5 shows the system performance comparison between ZF precoding and MF precoding for 100 transmit antennas. From Fig. 5, an increased 50 % UE throughput from 28 to 52 Mbps for ZF precoding can be observed. Another point to be noted here is that when non-cooperative massive MIMO is used, cell-edge users of MF precoding can have better system performance that those of ZF precoding because MF precoding also helps eliminating inter-cell interference, whereas for ZF precoding, inter-cell interference can only be treated as noise and cannot be eliminated.

C. *Channel Correlation*

Figure 6 shows the UE throughput CDF under different transmit correlation coefficients of 0.0, 0.2 and 0.5 respectively for 100 transmit antennas. From Fig. 6, as the transmit correlation coefficients increase from 0.0 to 0.2 to 0.5, a significant performance degradation (i.e., from 28 to 24 to 21 Mbps of 50 % UE throughput) can be observed, which shows that the channel correlations in practical systems may pose a substantial degradation to the system performance of massive MIMO.

D. *Imperfect Channel Estimation*

Figure 7 shows the UE throughput CDF under ideal and imperfect MMSE channel estimation respectively for 100 transmit antennas. From Fig. 7, a significant performance degradation (i.e., from 28 to 20 Mbps of 50 % UE throughput) can be observed, which shows that the imperfect channel estimation in practical systems may contribute a substantial degradation to the system performance of massive MIMO.

E. *Pilot Contamination*

Figures 4-7 don't take into account the pilot contamination effects on the system performance and Figure 8 shows the UE

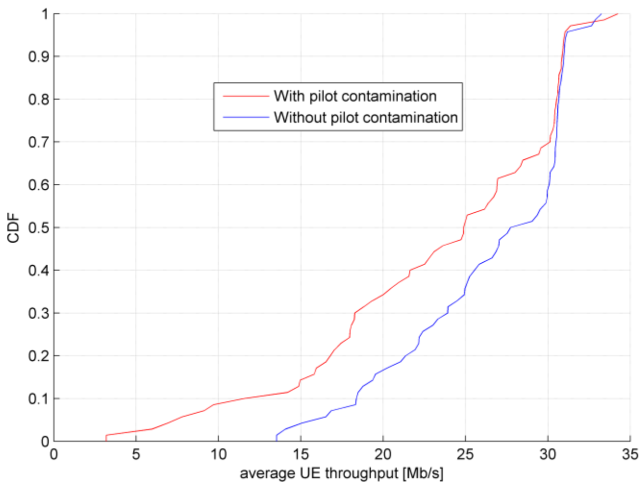

Fig.8. UE throughput CDF with/without pilot contamination for 100 transmit antennas

throughput CDF when pilot contamination effects are taken into account for 100 transmit antennas in the system performance. In the simulation, each cell use the same set of pilot resources as all other cells and we assume the number of pilot sequences is the same as the number of users in each cell. Each user in each cell randomly select one pilot sequence in the set, and all the users in the same cell use the orthogonal pilot sequences in the set It is found that further performance degradation will be incurred (i.e., from 28 Mbps to 25 Mbps of 50 % UE throughput) when considering pilot contamination effects.

V. Conclusions

In this paper, system level simulation performance of massive MIMO systems for downlink performance is presented based upon current LTE systems architecture and standards considering various basic performance limiting factors such as non-ideal asymptotical orthogonality, transmit channel correlation, imperfect channel estimation and pilot contamination. It is demonstrated that even as the number of transmit antennas is increased to a large number, the accumulative effects of the non-ideal asymptotical orthogonality between different users constitute a significant degradation to the system performance. Likewise, a significant performance degradation is also incurred when considering transmit channel correlation, imperfect channel estimation and pilot contamination. The system simulations presented in this paper provide a view of the potential system performance that can be achieved by massive MIMO technologies in practical 5G systems and a methodology to evaluate the achievable system performance of potential solutions of practical massive MIMO. Future research will be on system performance evaluation of various solutions to these limiting factors such as cooperative massive MIMO based upon 3D channel models and different antenna configurations.

VI. Acknowledgment

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